

# HadAtom99

Workshop on Hadronic Atoms<sup>1</sup>  
Institut für Theoretische Physik, Universität Bern  
Sidlerstrasse 5, CH-3012 Bern, Schweiz  
October 14 - 15, 1999

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## Abstract

These are the proceedings of the workshop "HadAtom99", held at the Institut für Theoretische Physik, Universität Bern, October 14 - 15, 1999. The main topics discussed at the workshop were the physics of hadronic atoms and in this context recent results from Chiral Perturbation Theory. Included here are the program, the list of participants and a short contribution from the speakers.

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# 1 Introduction

There has been a growth of interest in hadronic atoms in recent years. On the experimental side, energy levels of hadronic atoms as well as their decay into neutral hadrons have been or will be measured with high precision: Pionic hydrogen and  $\pi^-d$  atoms at PSI, ponium at CERN (DIRAC), and  $K^-p$  as well as  $K^-d$  atoms at DAFNE (DEAR). On the theoretical side, effective lagrangian techniques have shown to be very efficient also here: Bound state calculations – in particular the determination of energy levels and decay widths – can now be performed with surprising ease.

For these reasons, Leonid Nemenov suggested to organize a workshop on this subject at the Institut für Theoretische Physik, Universität Bern, similarly to the one organized at Dubna (Russia) in May 1998 on the same subject. The talks were devoted to recent experimental and theoretical progress in the investigations of hadronic atoms.

The meeting had 48 participants whose names, institutes and e-mail addresses are listed below. Results were presented in 25 minutes talks by 26 physicists. A one page abstract of these presentations as well as a list of the most relevant references has been provided by most of the speakers. These contributions are collected here. Below we display the list of participants with their e-mail, then follows the program of the workshop. Finally, we provide the individual excerpts submitted by the speakers <sup>2</sup>.

We would like to thank all participants for their effort to travel to Bern and for making this an exciting and lively meeting, and our secretaries, Ruth Bestgen and Ottilia Hänni, for their excellent support. Last but not least, we thank our colleagues from the organizing committee (Heinrich Leutwyler, Valery Lyubovitskij, Leonid Nemenov, Hagop Sazdjian, and Dirk Trautmann) for their invaluable help in structuring the meeting.

Bern, November 1999

Jürg Gasser, Akaki Rusetsky and Jürg Schacher

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<sup>2</sup> This kind of miniproceedings had been collected and posted on the archive in recent years by Hans Bijmans and Ulf Meißner at several occasions, see e.g. J. Bijmans and Ulf-G. Meißner, Chiral Effective Theories, 205. WE-Heraeus Seminar, Bad Honnef, November 30 - December 4, 1998, Miniproceedings, hep-ph/9901381. We found this an excellent way to summarize the event: it does not demand excessive labor from the authors, yet it provides a lot of useful information. Here we follow this idea.

## 2 List of participants

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### 3 HadAtom99: Program

THURSDAY, OCTOBER 14

10:00	-	10:05	J. Gasser (Bern)	Welcome
10:05	-	10:30	J. Schacher (Bern)	The DIRAC experiment at CERN
10:30	-	10:55	R. Lednicky (Prague)	Finite size effects on $\pi^+\pi^-$ production in continuous and discrete spectrum
10:55	-	11:20	A.V. Tarasov (Dubna)	Analytical approach to calculation of the $\pi^+\pi^-$ atom production rate in different quantum states
11:20	-	11:45	Coffee	
11:45	-	12:10	F. Ravndal (Oslo)	Corrections to the ponium lifetime
12:10	-	12:35	J. Soto (Barcelona)	Effective field theory approach to ponium
12:35	-	13:00	H. Sazdjian (Orsay)	Ponium lifetime in Generalized Chiral Perturbation Theory
13:00	-	14:30	Lunch	
14:30	-	14:55	V.E. Lyubovitskij (Dubna)	$\pi^+\pi^-$ atom in QCD
14:55	-	15:20	A. Rusetsky (Bern)	Spectrum and decays of hadronic atoms
15:20	-	15:45	L.G. Afanasyev (Dubna)	Calculation of the breakup probability of $\pi^+\pi^-$ atom in a target with a high accuracy
15:45	-	16:15	Coffee	
16:15	-	17:15	<div style="display: inline-block; vertical-align: middle;"> D. Trautmann (Basel)  Th. Heim (Basel)  K. Hencken (Basel) </div> <div style="display: inline-block; vertical-align: middle; font-size: 3em; margin: 0 10px;">}</div> <div style="display: inline-block; vertical-align: middle;"> <div style="display: inline-block; vertical-align: middle; font-size: 3em;">{</div> Group report  about ponium  interaction with matter </div>	
17:15	-	17:35	Coffee	
17:35	-	18:00	G. Baur (FZ Jülich)	Another exotic relativistic atom: antihydrogen
18:00	-	18:25	J. Cugnon (Liege)	Multiphoton exchange effects in ponium-matter interaction

FRIDAY, OCTOBER 15

9:00	-	9:25	L.L. Nemenov (CERN-Dubna)	Development in the theoretical meson description required by DIRAC experiment
9:25	-	9:50	C. Guaraldo (LNF-INFN)	The DEAR experiment at DAΦNE
9:50	-	10:15	L. Simons (PSI)	Pionic hydrogen: status and outlook
10:15	-	10:40	Coffee	
10:40	-	11:05	U.-G. Meißner (FZ Jülich)	Pion-kaon scattering
11:05	-	11:30	H. Leutwyler (Bern)	Discussion: pion-kaon scattering
11:30	-	11:45	Coffee	
11:45	-	12:10	P. de Simone (LNF-INFN)	$K_{l4}$ decays at DAΦNE
12:10	-	12:35	S. Pislak (BNL)	A new measurement of the $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay
12:35	-	13:00	G. Amoros (Lund)	$K_{\ell 4}$ decays: a theoretical discussion
13:00	-	14:30	Lunch	
14:30	-	14:55	J. Stern (Orsay)	Chiral phase transitions at zero temperature
14:55	-	15:20	L. Girlanda (Orsay)	The two-flavor chiral condensate from low-energy $\pi\pi$ phase-shifts
15:20	-	15:45	M. Knecht (Marseille)	Radiative corrections in $K_{l4}$ decays
15:45	-	16:15	Coffee	
16:15	-	16:40	G. Colangelo (Zurich)	Numerical solutions of Roy equations
16:40			J. Gasser (Bern)	Closing remarks

# The DIRAC Experiment at CERN

DIRAC Collaboration

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The experiment DIRAC [1], a magnetic double arm spectrometer, aims to measure the  $\pi^+\pi^-$  atom lifetime with 10% precision, using the high intensity 24 GeV/c proton beam of the CERN Proton Synchrotron. Since the value of this lifetime of order  $10^{-15}$  s is dictated by strong interaction at low energy, a precise measurement of this quantity enables to study characteristic pion parameters in detail and to submit predictions of QCD to a severe check.

After tuning the primary proton beam as well as all detector components in detail, the experiment started to accumulate data this summer. Preliminary results concerning quality and reliability of the setup are presented in the following two figures:

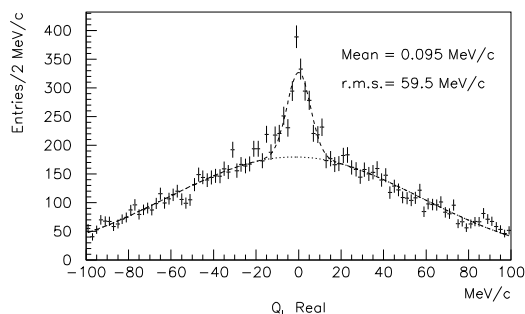


Figure 1: Correlated pairs of  $\pi^+$  and  $\pi^-$  with momenta  $p_{lab}$  smaller than 4.5 GeV/c and  $Q_T < 4$  MeV/c.

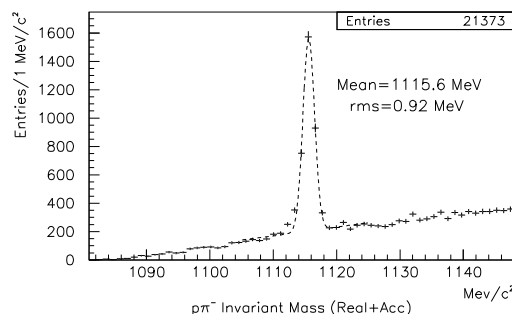


Figure 2: Invariant pair mass of  $\pi^-$  and p with momentum  $p_{lab}$  larger than 3 GeV/c.

In Fig. 1, the distribution of  $\pi^+\pi^-$  pairs in dependence of  $Q_L$  shows a noticeable peak in the yield of  $\pi^+\pi^-$  pairs with momenta  $Q$  less than 5 MeV/c, due to Coulomb attraction in the final state. The longitudinal component  $Q_L$  is defined as the projection of the relative momentum  $\vec{Q}$  in the  $\pi^+\pi^-$  system on the direction of the  $\pi^+\pi^-$  lab momentum. The mean value of the peak,  $\langle Q_L \rangle = 0.095$  MeV/c, is small and hence clear evidence for good setup alignment.

As emphasized in Ref. 1, the resolution in  $Q_L$  and  $Q_T$  plays a key role in the data analysis. In order to keep track of the relative momentum resolution, the invariant mass distribution of p and  $\pi^-$  — products of the  $\Lambda$  decay — was carefully investigated. The position of the mass peak in Fig. 2 corresponds to  $\Lambda$  particles, recorded and reconstructed in the magnetic spectrometer. The width of the  $\Lambda$  peak, mainly given by the momentum resolution, is found to be less than 1 MeV/c<sup>2</sup> (see Fig. 2). Thus, the DIRAC collaboration is confident, that the magnetic field and the location of the track detectors are known with adequate precision.

[1] B. Adeva *et al.*, Proposal to the SPSLC, CERN/SPSLC 95-1 (1995).

# Finite-size effects on $\pi^+\pi^-$ production in continuous and discrete spectrum

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The determination, on a per cent level accuracy, of the breakup probability  $P_{br} = N_A^{br}/N_A$  of the  $\pi^+\pi^-$  atoms produced by a high energy beam in a target is of principle importance for a precise lifetime measurement of these atoms in the experiment DIRAC [1] currently running at CERN. Clearly, the breakup probability is a unique function of the target geometry and material, the Lorentz factor and the ground state lifetime.

While the number of the breakup atoms  $N_A^{br}$  is measured directly as an excess of the  $\pi^+\pi^-$  pairs with very low relative momentum, the number  $N_A$  of the produced  $\pi^+\pi^-$  atoms is calculated based on the Migdal-Watson theory of final state interaction in discrete [2,3] and continuous [4] spectrum. This calculation is sensitive to the space-time extent of the pion production region, characterized by the distance  $r^*$  between the  $\pi^+$  and  $\pi^-$  production points in their c.m.s.. In ref. [2], the  $r^*$ -dependence was treated in an approximate way, dividing the pion sources into short-lived and long-lived ones. It was assumed that  $r^* = 0$  for pion pairs arising solely from the short-lived sources and characterized by the distances  $r^*$  much smaller than the Bohr radius of the  $\pi^+\pi^-$  system:  $|a| = 387.5$  fm, otherwise  $r^* = \infty$ .

The finite size correction to such calculated number of free  $\pi^+\pi^-$  pairs arises mainly from the region of small relative momenta compared with  $1/r^*$ . It is determined by the three dimensionless combinations  $r^*/a$ ,  $f_0/r^*$  and  $f_0/a$  of  $r^*$ ,  $a$  and the s-wave  $\pi^+\pi^-$ -scattering length:  $f_0 = \frac{1}{3}(2a_0 + a_2) \approx 0.23$  fm. Typically  $\langle r^* \rangle \sim 10$  fm so that the correction is dominated by the strong interaction effect and can amount up to  $\sim 10\%$ .

Due to a small binding energy  $\epsilon \sim (2\mu a^2)^{-1}$ , the finite-size correction to the discrete spectrum at  $r^* \ll |a|$  is nearly the same as that to the continuous spectrum at zero energy. Since  $N_A$  is actually determined by a ratio of the pions produced in discrete and continuous spectrum, this correction, up to a fraction of  $r^*/a$ , would cancel out in the breakup probability provided we could measure the number of free  $\pi^+\pi^-$  pairs in the region of very small relative momenta, much less than  $1/r^*$  [5,6]. With the increasing relative momentum the correction to the production of the free pairs decreases, the decrease being faster for pions emitted at larger distances  $r^*$ . Particularly important is the case when one of the two pions comes from the  $\omega$ -meson decay and  $r^* \sim 30$  fm. As a result, in DIRAC conditions, we expect the net finite size correction in the breakup probability of the order of per cent. It leads to several per cent effect in the measured lifetime and, in first approximation, can be neglected compared with the expected statistical error of  $\sim 10\%$ .

We discuss how to diminish the systematic error related to the finite size effects, using the correlation data on identical charged pions (containing the information about the distances  $r^*$  between the pion production points in the same experiment) together with the complete phase-space simulations within transport models. We also show that the usual equal-time approximation, the neglect of the space-time coherence as well as of the transition  $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$  and of the residual nucleus charge introduce negligible systematic errors of the order of per mill in the measured lifetime.

- [1] B. Adeva et al.: Lifetime measurement of  $\pi^+\pi^-$  atoms to test low energy QCD predictions, Proposal to the SPSLC, CERN/SPSLC 95-1, updated 10 November 1995.
- [2] L. Nemenov: *Yad. Fiz.* **41** (1985) 980.
- [3] V.L. Lyuboshitz: *Yad. Fiz.* **48** (1988) 1501 (*Sov. J. Nucl. Phys.* **48** (1988) 956).
- [4] R. Lednický, V.L. Lyuboshitz: *Yad. Fiz.* **35** (1982) 1316 (*Sov. J. Nucl. Phys.* **35** (1982) 770).
- [5] R. Lednický: On the breakup probability of  $\pi^+\pi^-$  atoms, DIRAC meeting, February, 1998; Finite-size effects on  $\pi^+\pi^-$  production in continuous and discrete spectrum, DIRAC meeting, February, 1999.
- [6] L. Afanasyev, O. Voskresenskaya: *Phys. Lett. B* **453** (1999) 302.

# Analytical approach to calculation of the $\pi^+\pi^-$ atom production rate in different quantum states

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A method of measurement of the  $\pi^+\pi^-$ -atom lifetime proposed by L.L.Nemenov [1] and which is under realization in the experiment DIRAC [2] at CERN Proton Synchrotron is essentially based on the assumption that the ratio  $R(nS/pS)$  of the  $\pi^+\pi^-$ -atom production rate in  $nS$ -states in hadron-nuclear interactions at high energy to the production rate of so the called “Coulomb”  $\pi^+\pi^-$  pairs (i.e. free  $\pi^+\pi^-$  pairs produced at a distance less or of the order of the  $\pi^+\pi^-$ -atom Bohr radius and having the relative momentum of an order of the Bohr momentum) can be calculated with accuracy better than 1%. Here we consider the accuracy of this ratio.

According to [1,3], the ratio can be written as:

$$R(ns/ps) = \frac{\left| \int M(\vec{r}) \psi_{ns}(\vec{r}) d^3r \right|^2}{\left| \int M(\vec{r}) \psi_{ps}(\vec{r}) d^3r \right|^2}, \quad (1)$$

where  $\psi_{ns}(\vec{r})$  is the  $\pi^+\pi^-$ -atom wave function with the principal quantum number  $n$  and zero orbital one ( $ns$ -states);  $\psi_{ps}(\vec{r})$  is the wave function of  $\pi^+\pi^-$  pair with the relative momentum  $p$  and with zero orbital momentum ( $s$ -wave);  $M(\vec{r})$  is the amplitude of  $\pi^+\pi^-$ -system production at the relative distance  $\vec{r}$ .

In ref.[4] it is shown that in the first order of the perturbation theory over the strong interaction  $R(nS/pS)$  is expressed via the a squared ratio of the well-known Coulomb wave functions.

Numerical calculations in the resent paper [5] show that the same ratio between discrete states is also expressed via the a squared ratio of the Coulomb wave functions at origin.

In this paper we analyse the influence of the strong interaction on the ratio  $R(nS/pS)$  analytically using a square well approximation for the strong interaction potential. It is shown that the result of ref.[4] is valid with accuracy of an order of ratio of the  $\pi^+\pi^-$ -scattering length to the  $\pi^+\pi^-$ -atom Bohr radius which is approximately equal to  $10^{-3}$ . Thus, both perturbative and nonperturbative calculations of  $R(nS/pS)$  confirm that Nemenov’s result [1] for this quantity is valid with the required accuracy.

[1] L.L. Nemenov, *Yad.Fiz.* **41** (1985) 980.

[2] B. Adeva et al., *Lifetime measurement of  $\pi^+\pi^-$  atoms to test low energy QCD predictions*, (Proposal to the SPSLC, CERN/SPSLC 95-1, SPSLC/P 284, Geneva 1995).

[3] L.G. Afanasyev, O.O. Voskresenskaya and V.V. Yazkov, *JINR Communication*, P1-97-306, Dunba 1997.

[4] L.G. Afanasyev and O.O. Voskresenskaya, *Physics Letters B* **453** (1999) 302. [5] I. Amirkhanov, I. Puzynin, A. Tarasov, O. Voskresenskaya and O. Zeinalova *Physics Letters B* **452** (1999) 1.



# Corrections to the ponium lifetime

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The charged pions with mass  $m_+$  in the 1S ground state of ponium are bound in the Coulomb wavefunction  $\Psi(r)$  with characteristic momentum  $\gamma = \alpha m_+/2$ . Its dominant decay mode is into two neutral pions with mass  $m_0$ . Since the mass difference  $\Delta m = m_+ - m_0$  is small, both the pions in the initial state of the hadronic atom and in the final state of the decay are non-relativistic. This was first emphasized by Labelle and Buckley [1] who showed that the leading term of the decay rate  $\Gamma$  can be obtained by calculating the energy shift  $\Delta E$  within non-relativistic effective field theory using the relation  $\Gamma = -2 \text{Im} \Delta E$ . The corresponding Lagrangian was then extended by Kong and Ravndal [2] to include higher order interactions which allow corrections to the dominant decay rate to be systematically calculated.

In order to determine the unknown coupling constants of the effective Lagrangian, one must match the non-relativistic scattering amplitude for  $\pi^+ + \pi^- \rightarrow \pi^0 + \pi^0$  from this theory to that which follows from relativistic, chiral perturbation theory at the same energy. At low momenta this latter one is defined to be

$$T_R(p) = 32\pi(a + bp^2/m_+^2) \quad (1)$$

in the center-of-mass frame where the charged pions have the momentum  $p$ . The scattering length  $a$  and the slope parameter  $b$  include both higher order chiral corrections and isospin-violating effects from the quark mass difference  $m_u - m_d$  and short-range electromagnetic effects. In the matching one must also include the  $p^2$  effects coming from the difference in normalization of relativistic and non-relativistic states as done correctly by Gall, Gasser, Lyubovitskij and Rusetsky [3].

At threshold the momentum  $p = 0$  and the full scattering amplitude with long-range Coulomb interactions removed is then given by just this scattering length. It can be written in terms of the more conventional isospin-symmetric scattering lengths  $a_0$  and  $a_2$  in the isospin  $I = 0$  and isospin  $I = 2$  channels as  $a = a_0 - a_2 + \Delta a$  where  $\Delta a$  includes these symmetry-breaking effects.

Calculating now the transition rate and multiplying by the probability  $|\Psi(0)|^2 = \gamma^3/\pi$  that the pions are found at the same point, one obtains the leading order result for the decay rate

$$\Gamma_0 = \frac{2}{9}\alpha^3 \frac{m_0}{m_+} \sqrt{2\Delta m m_0} a^2 \quad (2)$$

Relativistic effects in the final state can be obtained from a corresponding term of the effective Lagrangian [4] and follows also directly from the phase space factor

$$\Gamma \propto \int \frac{d^3k}{(2\pi)^3} \delta(2m_+ - 2E_0) \quad (3)$$

where  $E_0 = m_0 + k^2/2m_0 - k^4/8m_0^3$  is the energy of the neutral pions when we ignore the small binding energy. To this order in  $\Delta m/m$  the decay rate is thus found to be

$$\Gamma = \frac{2}{9}\alpha^3 \sqrt{2\Delta m m_0} \left(1 + \frac{\Delta m}{4m}\right) a^2 \quad (4)$$

This is consistent with the more general result of Gall *et al.* [3] and has also been found by Eiras and Soto [5].

Remaining corrections can now be obtained by including long-range Coulomb interactions and rescattering effects. These have been worked out by Gall *et al.* [3] and will also follow from the effective theory of Eiras and Soto [5]. The above non-relativistic approach can be similarly applied to other hadronic atoms.

[1] P. Labelle and K. Buckley, hep-ph/9804201.

[2] X. Kong and F. Ravndal, *Phys. Rev. D* **59**, 014031 (1999).

[3] A. Gall, J. Gasser, V.E. Lyubovitskij and A. Rusetsky, *Phys. Lett. B* **462**, 335 (1999).

[4] X. Kong and F. Ravndal, hep-ph/9905539.

[5] D. Eiras and J. Soto, hep-ph/9905543.

# Effective field theory approach to pionium

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I shall briefly report on the model-independent approach to pionium which has been presented in [1].

It basically consist of identifying the relevant energy and momentum scales of the  $\pi^+\pi^-$  atom and sequentially integrate them out until we reach the lowest relevant scale, namely the binding energy. This is carried out by introducing a series of non-relativistic effective field theories and by requiring them to be equivalent at the desired order of accuracy. Since pionium is an electromagnetic bound state, it contains at least three dynamical scales: the mass of the charged pions  $m \sim 140\text{MeV}$ , the typical relative momentum in the bound state  $m\alpha/2 \sim 0.5\text{MeV}$ , and the binding energy  $m\alpha^2/4 \sim 2\text{keV}$  [2]. In addition pionium decays mainly to two neutral pions through the strong interactions, which brings in an additional energy scale  $\Delta m \sim 5\text{MeV}$ , the difference between the charge and neutral pion masses, and its associated three momentum  $s = \sqrt{2m\Delta m} \sim 40\text{MeV}$ . In view of these numerical values two important observations are in order: (i) all the scales are considerably smaller than  $m \sim 140\text{ MeV}$ , which implies that a non-relativistic approach should be appropriated, and (ii) all the scales, including  $m$ , are considerably smaller than the typical hadronic scale, say the rho mass, and hence all the necessary information is contained in the Chiral Lagrangian ( $\chi\text{L}$ ) coupled to electromagnetism. Since at these energies not only electromagnetism but also the strong interactions are amenable to a perturbative treatment (in  $\alpha$  and in Chiral Perturbation Theory ( $\chi\text{PT}$ ) respectively), pionium can be addressed in a model independent way.

Let us then start with the  $SU(2) \times SU(2)$   $\chi\text{L}$  coupled to electromagnetism. After integrating out the scale  $m$  for pion pairs near threshold one should obtain what may be called a Non-Relativistic  $\chi\text{L}$  (NR $\chi\text{L}$ ). The degrees of freedom of NR $\chi\text{L}$  are photons and non-relativistic charged and neutral pion fields. NR $\chi\text{L}$  is analogous to Non-Relativistic QED [2], but contains non-relativistic pseudoscalar fields instead of Pauli spinor fields and includes the effect of the strong interactions. We present the general form of the NR $\chi\text{L}$  including isospin breaking terms. Since  $\Delta m \gg m\alpha/2, m\alpha^2/4$  we can also integrate out this scale and its associated three momentum  $s$  (neutral pions). We obtain a second Non-Relativistic  $\chi\text{L}$  which may be abbreviated as NR $\chi\text{L}'$ . Its degrees of freedom are photons and non-relativistic charged pions. The introduction of NR $\chi\text{L}'$  avoids having to solve eventually a coupled channel problem. Finally, integrating out photons of energy or momentum of the order  $m\alpha/2$  we obtain what may be called potential NR $\chi\text{L}'$  (pNR $\chi\text{L}'$ ), in analogy of potential NRQED [3]. At the order we are interested in, pNR $\chi\text{L}'$  contains non-relativistic charged pions interacting through a potential (electromagnetic and strong), and it is totally equivalent to a suitable quantum mechanical Hamiltonian.

A second order quantum mechanical perturbation theory calculation in pNR $\chi\text{L}'$  allows us to obtain the decay width and the spectrum at next to leading order in  $\Delta m/m, m\alpha^2/4\Delta m$  and  $\alpha$ . In order to match this precision a yet-to-be-done two loop calculation in the  $\chi\text{L}$  including photons is necessary.

The techniques described above may also be useful for other hadronic atoms.

Related work using non-relativistic effective field theories can be found in [4].

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# Pionium lifetime in generalized chiral perturbation theory

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The relationship between the pionium lifetime and the  $\pi\pi$  scattering lengths, including the sizable electromagnetic corrections, is analyzed in the framework of generalized chiral perturbation theory, in which the quark condensate value is left as a free parameter. The variation curve of the lifetime as a function of the combination  $(a_0^0 - a_0^2)$  of the  $S$ -wave scattering lengths is presented.

# $\pi^+\pi^-$ Atom in QCD

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The DIRAC experiment at CERN [1] aims to measure the lifetime of the  $\pi^+\pi^-$  atom in its ground state with high precision. The atom predominantly decays into two neutral pions. The measurement of the quantity  $\Gamma_{2\pi^0}$  will allow one to determine the difference  $|a_0 - a_2|$  of the strong  $S$ -wave  $\pi\pi$  scattering lengths with isospin  $I = 0, 2$  and to check the predictions for this quantity obtained in the standard version of ChPT [2], and to investigate the nature of spontaneous chiral symmetry breaking in QCD [3]. In order to perform this programme, the theoretical expression for the width must of course be known with a precision that matches the accuracy of the lifetime measurement of DIRAC.

We present a general expression for the corresponding decay width in the framework of QCD (including photons). This expression is proportional to the square of the relativistic  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  scattering amplitude  $\mathcal{A}$ , contains all terms at leading and next-to-leading order in isospin breaking, and is valid to all orders in the chiral expansion [4]:

$$\Gamma_{2\pi^0} = \frac{2}{9} \alpha^3 p^* \mathcal{A}^2 (1 + K), \quad (1)$$

where  $p^* = (M_{\pi^+}^2 - M_{\pi^0}^2 - \alpha^2 M_{\pi^+}^2/4)^{1/2}$ . The quantities  $\mathcal{A}$  and  $K$  are expanded in powers of  $\alpha$  and  $m_d - m_u$ . We count  $\alpha$  and  $(m_d - m_u)^2$  as small parameters of order  $\delta$ :

$$\begin{aligned} \mathcal{A} &= a_0 - a_2 + h_1 (m_d - m_u)^2 + h_2 \alpha + o(\delta), \\ K &= f_1 (m_d - m_u)^2 + f_2 \alpha \ln \alpha + f_3 \alpha + o(\delta), \end{aligned} \quad (2)$$

where the scattering lengths  $a_0, a_2$  and the coefficients  $h_i$  and  $f_i$  are evaluated in the isospin symmetry limit, i.e. at  $m_u = m_d = \hat{m}$  and  $M_\pi = M_{\pi^+} = 139.57$  MeV. The quantities  $h_i$  and  $f_i$  parameterize the corrections to the leading-order Deser's et al. type [5] formula. For the factor  $K$  we get the exact expression (without any chiral expansion)

$$K = \frac{\Delta_\pi}{9M_{\pi^+}^2} (a_0 + 2a_2)^2 + \frac{2\alpha}{3} (1 - \ln \alpha) (2a_0 + a_2) + o(\delta). \quad (3)$$

Therefore, the formula (1) relegates the problem of the calculation of the  $\pi^+\pi^-$  atom lifetime to the evaluation of the physical on-mass-shell scattering amplitude for the process  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  to any desired order in the chiral expansion.

Next we apply Chiral Perturbation Theory at one loop to analyse the formula for the  $\pi^+\pi^-$  atom lifetime (1). We obtain the analytic expressions for the next-to-leading order  $O(\alpha)$  and  $O(m_d - m_u)^2$  corrections and pin them down numerically.

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# Spectrum and decays of hadronic atoms

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A general survey of the theoretical approaches to the description of hadronic atom characteristics (spectrum and decay width) is given. We confront nonrelativistic effective Lagrangian approach, as well as other field-theoretical approaches based on relativistic equations, with the potential scattering theory formalism for the hadronic atom problem. This comparison is essential in the view of several ongoing or planned experiments (DIRAC at CERN, PSI, DEAR at LNF-INFN, KEK, Uppsala) that are aimed at the high-precision measurement of hadronic atom characteristics. These measurements will enable one to directly extract the strong scattering amplitudes from the data, and thus contribute a valuable piece of information to our knowledge of the dynamics of strong interactions at low energy.

The analysis of the data in the above-mentioned experiments are carried out with the use of the relations given in Ref. [1]. Up to the corrections in the next order in isospin breaking, these relate the energy-level shift of the hadronic atom  $\Delta E$  and its partial decay width into the channel with neutral isotopic partners, to the certain isotopic combinations of the strong scattering lengths. However, in order to fully exploit the high-precision data available from recent and future experiments on hadronic atoms, one has to consistently evaluate the corrections to the relations given in Ref. [1]. Using the powerful technique based on the effective nonrelativistic Lagrangians, it is demonstrated that in the next-to-leading order in isospin breaking, and in all orders in chiral expansion, the partial decay width of  $\pi^+\pi^-$  atom into  $2\pi^0$  is given by [2]

$$\begin{aligned}\Gamma_{2\pi^0} &= \frac{2}{9} \alpha^3 (M_{\pi^+}^2 - M_{\pi^0}^2 - \frac{1}{4} \alpha^2 M_{\pi^+}^2)^{1/2} \mathcal{A}^2 (1 + K) \\ K &= \frac{\Delta_\pi}{9M_{\pi^+}^2} (a_0 + 2a_2)^2 + \frac{2\alpha}{3} (1 - \ln \alpha) (2a_0 + a_2)\end{aligned}\quad (1)$$

and  $\mathcal{A}$  stands for the regular part of *physical* scattering amplitude for  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  at threshold, calculated at the leading order in isospin breaking, and in all orders in chiral expansion. The strong energy-level shift of the  $\pi^-p$  atom is also expressed solely in terms of the regular part of *physical*  $\pi^-p \rightarrow \pi^-p$  scattering amplitude at threshold, calculated at the leading order in isospin breaking. The relations between the observables of hadronic atoms and the physical scattering amplitudes obtained by using the nonrelativistic Lagrangian technique, provide a desirable generalization of the relations from Ref. [1] that now contains *all* leading-order isospin-breaking corrections. These relations are obtained under a very general assumptions about the mass differences of isotopic partners, and do not resort explicitly to the chiral expansion of the corresponding strong amplitudes. In order to extract the strong scattering lengths from the physical scattering amplitudes that are actually measured in the experiment, one has, however, to use chiral expansion for these amplitudes, and establish such relations in a given order in chiral expansion.

By using chiral expansion, it is demonstrated that the potential scattering theory approach in its present form does not account for all isospin-breaking corrections to the strong amplitude. The dominant isospin-breaking contributions which are neglected in the potential approach, have been explicitly identified. First of all, these are effects of direct quark-photon coupling, that are encoded in the "electromagnetic" counterterms of the chiral Lagrangian. Then, there are effects coming from "tuning" of the quark mass so that the common mass of the pion triplet in the isospin-symmetric world coincides, by definition, with the charged pion mass. We conclude, that for comparison to the results of potential approach, it is necessary to match the latter with ChPT in the isospin-broken phase, in order to consistently take into account above effects.

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# Calculation of the breakup probability of $\pi^+\pi^-$ atom in a target with a high accuracy

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The interaction of  $\pi^+\pi^-$ -atom ( $A_{2\pi}$ ) with matter is of great importance for the DIRAC experiment [1] as the  $A_{2\pi}$  breakup (ionization) in such interactions is exploited to observe  $A_{2\pi}$  and to measure its lifetime. In the experiment the ratio of the number of  $\pi^+\pi^-$ -pair from the  $A_{2\pi}$  breakup inside a target to the number of produced  $A_{2\pi}$  (called the probability of  $A_{2\pi}$  breakup) will be measured. The measurement of the  $A_{2\pi}$  lifetime is based on the comparison of this experimental value with its calculated dependency on the lifetime. So the accuracy of this calculation is essential for the extraction of the lifetime.

In the DIRAC proposal the probability of  $A_{2\pi}$  breakup was calculated by evaluating of a set of differential equations for populations of  $A_{2\pi}$  states [2]. These equations describe the evolution of the populations, when  $A_{2\pi}$  traverses the target, for all atomic states with the principal quantum number less than some limit. The initial condition for this set of equations is given by the probability of the  $A_{2\pi}$  production in various quantum states [3]. As a solution we can get three probabilities for the case behind the target: 1) the summed population  $P_{\text{dsc}}$  of all discrete states considered in this set; 2) an estimate for the summed population of all other discrete states  $P_{\text{tail}}$ ; 3) the probability of  $A_{2\pi}$  annihilation  $P_{\text{anh}}$ . The remainder is the probability of the  $A_{2\pi}$  breakup  $P_{\text{br}}$ :

$$P_{\text{br}} = 1 - P_{\text{dsc}} - P_{\text{tail}} - P_{\text{anh}}.$$

The accuracy of this procedure is mainly defined by the accuracy of the cross sections for the  $A_{2\pi}$  interaction and target atoms. Now approaches are available to calculate these cross sections in the Glauber approximation [4] which takes into account all multi-photon exchanges instead of only the single-photon exchange in the first Born approximation as used before. This could provide an accuracy in the cross sections at the level less than 1% and hence almost the same accuracy for  $P_{\text{br}}$ .

To estimate a limit in the accuracy of the  $P_{\text{br}}$  calculation based on probabilities we have repeated all our calculation of ref.[2] using the parabolic basis for the  $A_{2\pi}$  states description instead of the spherical one as in [2]. In both cases we neglect interference effects in the  $A_{2\pi}$  description but its contribution should be different for another basis. So, the difference in  $P_{\text{br}}$  obtained with two bases shows a principal limit in accuracy for this method. In the following table the results of the calculation are shown for the  $A_{2\pi}$  momentum 4.7 GeV/c, the lifetime  $3.0 \cdot 10^{-15}$  s and target thicknesses equivalent in multiple scattering to 30  $\mu\text{m}$  Ta.

	Z	$P_{\text{br}}^{\text{sph}}$	$P_{\text{br}}^{\text{par}}$	$\Delta P_{\text{br}}/P_{\text{br}}$	$\Delta\tau/\tau$
Al	13	0.223	0.229	0.0292	0.079
Ti	22	0.326	0.330	0.0125	0.030
Fe	26	0.435	0.438	0.0069	0.018
Ni	28	0.470	0.473	0.0059	0.017
Mo	42	0.540	0.543	0.0044	0.015
Ta	73	0.671	0.673	0.0026	0.015
Pt	78	0.704	0.706	0.0022	0.017

The value of  $\Delta\tau/\tau$  could be considered as the bias of the method. To go lower than 1% we need to consider the process of the  $A_{2\pi}$  passage through the target in terms of amplitudes instead of probabilities.

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# Pionium interacting with matter:

## I. Formalism for coherent interaction

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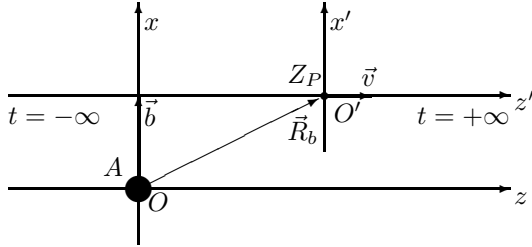
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The experiment DIRAC, currently being performed at CERN [1], aims at measuring the lifetime for pionium, i.e.  $\pi^+\pi^-$ -atoms, in its ground state to better than 10% accuracy thus providing a crucial test for chiral perturbation theory. Attaining the required precision hinges on an accurate calculation of all electromagnetic processes, competing with the strong interaction, to a precision of a few percent. Therefore we have applied the semi-classical formalism (SCA) to calculate the electromagnetic excitation and ionization, i.e. breakup, of the pionium to such high precision.

The SCA has been established [2–4] as a powerful tool to investigate such excitation processes even at relativistic energies. In this framework we describe an arbitrary collision process in terms of a trajectory associated with the projectile and characterized by an impact parameter  $b$ . The amplitude for the transition—excitation or ionization—of the target system from an initial state  $|i\rangle$  to a final state  $|f\rangle$  is given in first order of the interaction by the matrix element

$$a_{fi}^{(1)}(b) = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle f | H_{\text{int}}(\vec{R}_b(t)) | i \rangle, \quad (1)$$

where  $\vec{R}_b(t)$  describes the projectile's trajectory specified by the impact parameter in the target's rest frame (see figure).



**Figure:** Semi-classical picture of a projectile with charge  $Z_P$  moving on a trajectory with impact parameter  $b$  past a target atom  $A$  consisting of particles of opposite charge.  $\vec{R}_b(t)$  describes the projectile's trajectory specified by the impact parameter in the target's rest frame.

In our case, in the rest-frame of the pionium, the target material's ion can be treated as a classical particle moving on a straight-line trajectory  $\vec{R}_b(t) = (b, 0, \beta ct)$  at nearly the speed of light, while the pionium at the origin is treated quantum mechanically. As we are only interested in pions forming atom-like complexes, their relative velocity must be small (of order  $v_\pi/c \approx \alpha$ ). Hence non-relativistic hydrogenic wave functions are perfectly appropriate for the initial and final states  $|i\rangle$  and  $|f\rangle$  of the pionium. On the other hand, the complex charge distribution of the target atoms is taken into account by including a screening function in the scalar potential experienced by the pionium. In our calculations, we use screening functions that reproduce exactly expectation values of powers of the radial variable obtained with full Dirac-Hartree-Fock-Slater wave functions for the heavy ion or atom [5, 6]. Therefore by neglecting magnetic terms (estimated to contribute no more than 0.4%), the interaction Hamiltonian  $H_{\text{int}}$  reduces to the scalar (Liénard-Wiechert-) potential between the ion with charge  $Z_P$  and the pionium. In the pionium's rest frame we thus have:

$$\Phi(\vec{r}, t) = \frac{Z_P e}{2\pi^2} \sum_{k=1}^N A_k \int \frac{\exp[i\vec{s} \cdot (\vec{r} - \vec{R}_b(t))]}{s^2 + \alpha_k^2 - (\beta s_z)^2} d^3 s, \quad (2)$$

where the screening parameters  $A_k$  and  $\alpha_k$  are taken from [6]. Performing the integration over coordinate space implied in the matrix element (1) with the two pions positioned at  $\pm \vec{r}/2$  from the pionium's center-of-mass, we then obtain the impact parameter dependent transition amplitude in first order of the scalar interaction:

$$\begin{aligned} a_{fi}^{(1)}(b) &= \frac{2Z_P \alpha}{i\beta} \sqrt{4\pi(2\ell_f + 1)(2\ell_i + 1)} (-1)^{m_f} \sum_{\ell, m} i^{\ell-m} \sqrt{2\ell+1} \begin{pmatrix} \ell_f & \ell & \ell_i \\ 0 & 0 & 0 \end{pmatrix} \\ &\times [1 - (-1)^\ell] \begin{pmatrix} \ell_f & \ell & \ell_i \\ -m_f & m & m_i \end{pmatrix} \sum_{k=1}^N A_k \int_0^\infty s ds \frac{B_{\ell m}(b, q_0, s)}{s^2 + \alpha_k^2 - (q_0 \beta)^2} F_{fi}^\ell\left(\frac{s}{2}\right), \end{aligned} \quad (3)$$

with  $q_0 = (E_f - E_i)/(\beta\hbar c)$ , the straight-line trajectory factor  $B_{\ell m}(b, q_0, s)$  [7], and the radial form factors  $F_{fi}^\ell(k)$ , depending on the bound state or continuum wave functions of the ponium, as appropriate. These form factors can easily be evaluated using standard methods as described thoroughly in [8] and [9].

Integrating the squared amplitude over the impact parameter  $b$  yields the inelastic cross section  $\sigma_{fi}^{(1)}$  for the transition between the states  $|i\rangle$  and  $|f\rangle$  and the total cross section is then obtained by summing over bound and integrating over continuum final states—or by using the completeness relation for the set of final states:

$$\sigma_{\text{tot},i}^{(1)} = \sum_f \sigma_{fi}^{(1)} = 16\pi \left( \frac{Z_P \alpha}{\beta} \right)^2 \int_{q_0}^{\infty} s \, ds [1 - F_{ii}^0(s)] \left[ \sum_{k=1}^N \frac{A_k}{s^2 + \alpha_k^2 - (\beta q_0)^2} \right]^2. \quad (4)$$

The determination of transition probabilities and cross sections thus reduces to the accurate and fast calculation of the radial form factors.

In [8] we have shown the suitability of our approach under quite general conditions. We have calculated the total inelastic cross section in dependence of the kinetic energy of the heavy ion (in the rest frame of the ponium) for different target ions. Hereby we have shown that the total cross section, divided by the dominating (but trivial) factor  $1/\beta^2$  in (4), is essentially constant in the energy range of interest to experiment DIRAC, i.e., between 2 GeV and 10 GeV. Furthermore we have calculated cross sections for various transitions and break-up processes also in the screened Coulomb field of a nucleus both analytically as well as numerically.

Our on-going developments now concentrate on including magnetic interaction terms, as well as all higher-order perturbation contributions.

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## Pionium interacting with matter: II. Formalism for incoherent interaction

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The atomic structure was included in the previous calculation only via the (elastic) form factor  $F(q^2)$  [1]. Of course the (electrons of the) atoms can be excited as well. A simple estimate is that these incoherent/inelastic processes are of the order  $1/Z$  compared to the elastic/coherent one; therefore, in order to achieve the desired 1% accuracy needed for the experiment, a more precise calculation is needed.

We derive the cross section for the incoherent contribution starting from PWBA (Plane Wave Born Approximation) [2]. The electromagnetic transitions of the atoms and the Pionium are given through the electromagnetic tensor  $W^{\mu\nu}$ , which due to gauge invariance and current conservation, only depends on two scalar structure functions  $W^{(1)}(q^2, qP)$  and  $W^{(2)}(q^2, qP)$ . This is of course well known, e.g., from electron scattering [3]. Taking into account only the scalar interaction and neglecting the magnetic terms, only  $W^{(2)}$  is important and is proportional to the square of the charge transition. One advantage of this approach is that even though the relative motion of atom and Pionium is relativistic, the structure functions  $W$  can be calculated in the individual rest frames and therefore nonrelativistic approximations can be made.

We neglect recoil corrections and find the total cross section to be

$$\sigma = \int d\omega d\Delta d^2q_{\perp} \frac{4\alpha^2}{\beta^2} \frac{W_{2\Pi}(\omega, q^2) W_{2A}(\Delta, q^2)}{(q^2)^2}$$

with  $\omega$  the excitation energy of the Pionium (in its rest frame) and  $\Delta$  the one for the atom. The momentum square of the photon is given by

$$q^2 = - \left( \frac{\Delta^2}{\beta^2 \gamma^2} + \frac{\omega^2}{\beta^2 \gamma^2} + \frac{2\omega\Delta}{\beta^2 \gamma} + q_{\perp}^2 \right) = - (q_l^2 + q_{\perp}^2)$$

The relative Lorentzfactor is given by  $\gamma$ ,  $\beta = v/c$ .

We discuss approximations made to find closed expressions for the total cross section. In them the  $\omega$  and  $\Delta$  dependence in  $q^2$  are replaced by average values. We also find the cross section in the elastic case to be identical to the one from the SCA calculation, see previous contribution.

To get the total incoherent contribution, we sum over all possible excitations of the atom. This so-called inelastic scattering function  $S_{inc}$  is then given by

$$S_{inc}(k) = \int d\Delta W_2(\Delta, k^2) = \sum_{X \neq 0} |F_{X0}(k)|^2$$

with  $F_{X0}$  the transition form factor. For the calculation we make use of mean-field wave functions calculated within a self consistent Dirac-Hartree-Fock-Slater calculation [4]. In terms of the single electron wave functions  $S_{inc}$  is then given by

$$S_{inc}(\vec{k}) = Z - \sum_{j=1}^Z \sum_{l=1}^Z |\langle \Phi_j | \exp(i\vec{k} \cdot \vec{r}) | \Phi_l \rangle|^2.$$

The often used “no correlation limit” for  $S_{inc}$  is found to be not accurate enough. This is due to the fact that at the small  $q^2$ , important in our case, atoms are mainly excited and not ionized; therefore Pauli blocking, which is neglected in the no-correlation limit, is important.

Results of calculations within this formalism are presented in the next contribution.

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# Pionium interacting with matter: III. Results

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We present results of a detailed investigation of the target-elastic and target-inelastic electromagnetic cross sections for pionium scattering off various target materials. Within the closure approximation [1] the total electromagnetic cross sections from a given pionium initial state  $i$  can be written as

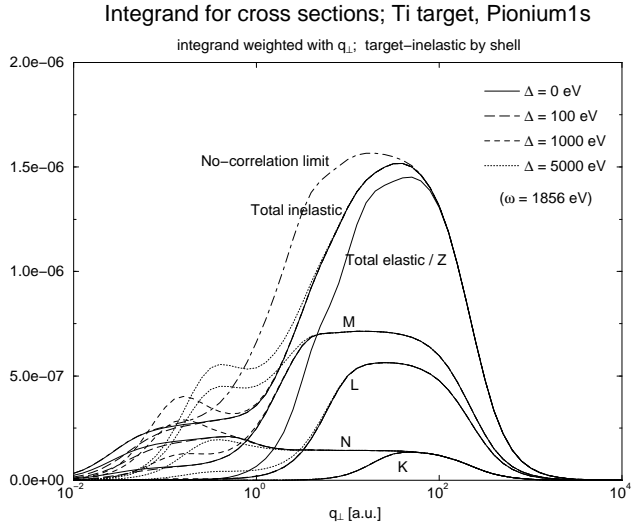
$$\sigma_{\text{tot}} = 16\pi \left(\frac{\alpha}{\beta}\right)^2 \int_0^\infty dq_\perp q_\perp \frac{\Phi_A(k)}{q^4} (1 - f_{ii}^0(s)) \quad (1)$$

where the momentum transfer variables  $s$  and  $k$  refer to the rest frame of the pionium and the atom, respectively.  $f_{ii}^0$  denotes the monopole formfactor for the pionium in the state  $i$ . For the target-elastic total cross section the atomic structure is given by  $\Phi_A(k) = (Z - F_{00}(k))^2$ , where  $F_{00}$  denotes the coherent formfactor of the electronic ground state orbitals only, as the contribution from the (point-like) nucleus has been separated out. We calculate the atomic formfactor in the framework of the Dirac-Hartree-Fock-Slater model, i.e., using orbitals obtained from the numerical solution of the Dirac (or Schrödinger) equation for each occupied orbital. However, the coherent formfactor is very well approximated by simple analytical expressions as given by Molière or Salvat *et al.* [2].

In the case of the target-inelastic total cross section, the atomic structure is contained in  $\Phi_A(k) = S_{\text{inc}}(k)$  as given in the preceding contribution. We determine  $S_{\text{inc}}$  using the same orbitals as for the coherent formfactor. In a simple “no-correlation limit”,  $S_{\text{inc}}(k) = Z - [F_{00}(k)]^2/Z$ . This approximation is *not* sufficiently accurate, as it completely neglects Pauli blocking and thus over-estimates the cross section. Our approach with Dirac-Hartree-Fock orbitals correctly accounts for Pauli blocking in  $S_{\text{inc}}(k)$ . Moreover it affords determining the contributions to the pionium cross sections for each atomic shell individually. This information cannot be extracted from tabulated values of formfactors and scattering functions [3] where only the combined contributions of all atomic shells are given.

The diagram on the right shows the integrand of (1), multiplied with the integration variable  $q_\perp$  (thus representing more clearly the relative magnitude of the individual contributions to the integral on a logarithmic scale for  $q_\perp$ ).

The target material is Ti ( $Z = 22$ ), pionium is initially in its ground state. Note that the choice of atomic excitation energy  $\Delta$  affects the integrand only at  $q_\perp$  smaller than the range of dominating contributions. The main contributions to the integral come from a complex interplay of photon propagator, atomic, and pionium structure at  $1 \leq q_\perp \leq 100$  a.u. Note also that the outer shells contribute much more to the target-inelastic cross section than the inner shells. Not only are there more electrons in outer shells than in inner shells, but the contribution of each individual target electron is approximately proportional to its principal quantum number. Finally, note that the target-inelastic cross section is significantly larger than the target-elastic divided by  $Z$ . Thus neither the “no-correlation” limit, nor the simple scaling approximation  $\sigma_{\text{inc}} \approx \sigma_{\text{coh}}/Z$  are accurate enough.



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# Another exotic relativistic atom: antihydrogen

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It is certainly difficult to do experiments with fast neutral atoms. Recently, antihydrogen has been produced and detected at LEAR/CERN [1] and Fermilab [2]. Cross-sections for the process

$$Z + \bar{p} \rightarrow Z + \overline{H_0} + e^- \quad (1)$$

are calculated [3], see also figure below. The formalism is very similar to the one for calculating the breakup of Pionium in matter, see also these miniproceedings. Lamb-shift measurements of hydrogen [4] and antihydrogen in flight, which are planned at Fermilab are briefly discussed, see Refs. [4,5,6].

Maybe related experimental techniques could also be relevant for the measurement of the Lamb-shift in Pionium. Of course, the properties of these atoms have very different scales (lifetime, size, mass, ...). This 2s-2p energy splitting is an interesting quantity since the scattering lengths  $a_0$  and  $a_2$  are involved in a different combination as compared to the one, which enters in the Pionium lifetime [7].

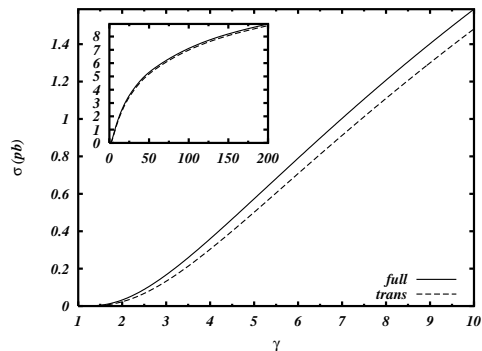


Fig. 1: Cross sections for the reaction eq. 1 for  $Z = 1$ . The Lorentz factor  $\gamma$  of the incoming antiproton in the Fermilab experiment [2] was in the range of 5.6–6.7, corresponding to an antiproton momentum range of 5200–6200 MeV/c. A cross section of  $\sigma_{1s+2s} = (1.12 \pm 0.14 \pm 0.09)\text{pb}$  was found. (The cross section  $\sigma_{ns}$  for capture into the  $s$ -state with principal quantum number  $n$  scales as  $1/n^3$ .)

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# Multiphoton exchange in ponium-matter interaction

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The interaction of a high velocity ponium atom colliding with an ordinary atom is investigated by studying the evolution of the internal wave function driven by the time-dependent Coulomb field generated by the ordinary atom. This evolution is governed by the following time-dependent equation:

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} (H_0 + V(t)) \psi, \quad (1)$$

where  $H_0$  is the intrinsic ponium Hamiltonian and where  $V(t)$  is the Coulomb interaction between the ponium and the external atom. Using the expansion of  $\psi$  in terms of the eigenstates  $\psi_i$  of  $H_0$ , i.e.

$$\psi = \sum_i c_i e^{-\frac{i}{\hbar} \epsilon_i t} \psi_i, \quad (2)$$

one obtains a set of coupled linear differential equations for the coefficients  $c_i$ :

$$\frac{dc_i}{dt} = -\frac{i}{\hbar} \sum_j V_{ij}(t) c_j(t). \quad (3)$$

Here, we solve this set of equations in a subset of bound ponium states. We assume that the atom is moving along a straight line with a constant velocity, in the rest frame of the ponium. We adopt the dipole approximation, i.e. that the Coulomb interaction is reduced to the interaction between the electric dipole of the ponium and the electric field generated by the atom. For the latter, the Molière form factor is used. Equations (3) are integrated with proper initial conditions  $c_j(t=0) = \delta_{ij}$ , corresponding to the ponium being in state  $\psi_i$  initially. The cross section for the excitation from state  $\psi_i$  to state  $\psi_j$  is obtained by integrating over the impact parameter  $b$

$$\sigma_{i \rightarrow j} = \int_0^\infty db \, b \, |c_j(t \rightarrow \infty)|^2. \quad (4)$$

Keeping the initial values  $c_j = \delta_{ij}$  in the r.h.s. of the equations (3) yields the usual Born approximation[1]. We also show that expression (4) with Born approximation is equivalent to the field theoretical expression for one photon exchange, as obtained in ref.[2], when recoil correction is neglected, provided a one-to-one correspondance is introduced between the impact parameter and the tranverse momentum transfer. This allowed us also to correct the dipole approximation used in  $V(t)$  at small impact parameter (details can be found in ref.[3]). The difference between the exact solution of Eq. (3) and the Born approximation can thus be interpreted as due to multiphoton exchanges.

This contribution has been evaluated numerically for a few illustrative cases. The dependance upon the impact parameter, the atomic number of the external atom and the incident energy is investigated. For the most intense transition  $1S \rightarrow 2P$ , the effect of multiphoton exchange is a reduction of the transition cross section. This reduction is effective for reduced impact parameter  $\tilde{b}$  ( $= b/a$ ,  $a$  being the ponium Bohr radius) lying between  $\sim 0.5$  and  $\sim 2$ . Let us mention that the bulk of the cross section involves  $\tilde{b}$  between  $\sim 0.5$  and  $\sim 5$  and that the dipole approximation is only effective for  $\tilde{b}$  less than unity.

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# The DEAR experiment at DAΦNE

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A new era in the field of low energy kaon physics will begin with the start of the new DAΦNE low-energy, high-luminosity  $e^+e^-$  collider. The objective of the **DEAR** (**DAΦNE Exotic Atom Research**) experiment is the determination of the isospin dependent  $\bar{K}N$  scattering lengths via the measurement of the strong interaction shifts and widths of the kaonic hydrogen and kaonic deuterium  $K$ -series lines [1]. In practice, in the case of kaonic hydrogen, the  $K_\alpha$  line at  $\sim 6.15$  keV is the most important one for the determination of the shift and width of the  $1s$  level.

The challenging goal DEAR is aiming at is to measure the  $K_\alpha$  line shift in hydrogen to a precision of 1 %, the width to a precision of a few percent, and kaonic deuterium for the first time.

The main challenge of DEAR is to detect a very weak x-ray signal out of the background of an  $e^+e^-$  collider with high statistics and high precision. Only recently these X rays have been observed for the first time (KpX at KEK [2]) with an overall statistics of only 114 counts in the  $K_\alpha$  peak.

To overcome the background problem Charge-Coupled Devices (CCDs) are used as x-ray detectors. CCDs are characterized by a good energy resolution and by an unprecedented background rejection capability [3].

A kaonic atom is formed when a negative kaon enters a target, loses its kinetic energy through ionization and excitation of the molecules of the medium and eventually is captured, replacing the electron, in an excited orbit. Various cascade processes deexcite this kaonic atom to the ground state.

When a kaon reaches low- $n$  states with small angular momentum, it is absorbed through a strong interaction with the nucleus. This strong interaction causes a shift in the energies of the low-lying levels from their purely electromagnetic values, whilst the finite lifetime of the state turns out in an increase in the observed level width.

The shift  $\epsilon$  and the width  $\Gamma$  of the  $1s$  state of kaonic hydrogen are related in a fairly model-independent way to the real and imaginary part of the complex  $s$ -wave scattering length,  $a_{K^-p}$ :  $\epsilon + \frac{i}{2}\Gamma = 2\alpha^3\mu^2 a_{K^-p} = (412 \text{ eVfm}^{-1}) \cdot a_{K^-p}$  (Deser-Trueman formula [4]), where  $\alpha$  is the fine structure constant and  $\mu$  the reduced mass of the  $K^-p$  system. A similar relation applies to the case of kaonic deuterium and the corresponding scattering length,  $a_{K^-d}$ .

These observable scattering lengths are related to the isospin dependent scattering lengths  $a_0$  and  $a_1$  in the following way:  $a_{K^-p} = \frac{1}{2}(a_0 + a_1)$  ,  $a_{K^-d} = \frac{1}{2} \left( \frac{m_N + m_K}{m_N + m_K/2} \right) (a_0 + 3a_1) + C$  . In the case of deuterium, the first term represents the lowest-order impulse approximation, in which the kaon scatters on each “free” nucleon. The second term,  $C$ , contains all high-order contributions, including three-body effects. This term can be larger than the first term, thus the extraction of the two isospin scattering lengths from an observation of the kaonic hydrogen and kaonic deuterium scattering lengths requires a dedicated analysis. An accurate determination of the  $K^-N$  isospin dependent scattering lengths will place strong constraints on low energy  $K^-N$  dynamics, which in turn constraints the SU(3) description of chiral symmetry breaking.

Crucial information about the nature of chiral symmetry breaking, and to what extent chiral symmetry is broken, is provided by the calculation of the meson-nucleon sigma terms. The meson-nucleon sigma terms are defined as the expectation value of a double commutator of the chiral symmetry breaking part of the strong-interaction Hamiltonian. The low energy theorem relates the sigma terms to the meson-nucleon scattering amplitude. A phenomenological procedure starting from the experimental amplitudes is then used to determine the sigma terms and therefore measure chiral symmetry breaking. According to an evaluation based on the uncertainties in the phenomenological procedure the sigma terms can be extracted at the level of 20 %, by combining the precision measurement to be performed by DEAR with the bulk of most recent analyses of low energy  $K^\pm N$  scattering data. The sigma terms are also important inputs for the determination of the strangeness content of the proton.

Presently, the DEAR experiment is installed on DAΦNE and ready to start data taking.

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# Pionic hydrogen: status and outlook.

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The measurement of the strong interaction shift and width of the ground state of the pionic hydrogen atom allows to determine two different linear combinations of the two isospin separated s-wave scattering lengths of the pion nucleon system. Past experiments at PSI measured the 3-1 transition energy in pionic hydrogen and deuterium [1,2,3]. The measurement combined a highly efficient stopping arrangement (cyclotron trap) and a sophisticated cylindrically bent Bragg crystal set-up with newly developed CCD detectors. The shift was finally determined with an accuracy of better than  $10^{-2}$ . The error in the determination of the width of the ground state level in hydrogen is a factor of almost one order of magnitude worse than the error in the shift value, which inhibits a determination of both isospin separated scattering lengths on the percent level from the pionic hydrogen measurement alone. The situation can be improved in a combined analysis with the pionic deuterium shift measurement, but then one has to rely on a theoretical understanding of the pionic deuterium system.

A new proposal at PSI aims at a direct determination of the width of the ground state of pionic hydrogen on the level of  $10^{-2}$  [4]. In order to achieve this the experiment should meet several requirements as are a higher number of detected X-rays and a thorough understanding of the response function of the crystal spectrometer. Most importantly it should also be able to disentangle the strong interaction broadening from other line broadening effects as the Doppler effect. Pionic hydrogen atoms may gain kinetic energy by converting binding energy difference into kinetic energy, a process known as Coulomb deexcitation. A recent experimental investigation of this effect took place at PSI. The distortion of neutron time of flight spectra from the charge exchange reaction has been measured in liquid hydrogen and in gas at a pressure of 40 bar (room temperature) [5]. The measured spectra clearly show the effect of different transition steps converted into kinetic energy.

The statistics problem was solved by stopping more pions in a newly built cyclotron trap and by increasing the efficiency of the crystal spectrometer by using spherically bent crystals as well as bigger CCD detectors. In this way a factor of more than one order of magnitude was gained in count rate. In addition the peak to background ratio as well as the resolution of the spectrometer had been improved. First experiments with pionic deuterium show the high quality of the data achieved by these measures [6].

For a better determination of the response function an ECR (Electron Cyclotron Resonance) source is presently being set up which produces X-rays from hydrogen-like ions with high intensity. Their line widths are expected to be an order of magnitude narrower than the expected resolution of the spectrometer. Under these circumstances a detailed measurement of the response function and even an optimization of the crystals will be feasible.

The third problem will be attacked in two steps. First the 2-1 and the 3-1 transitions in pionic hydrogen will be measured at about three different pressures. The influence of the Doppler effect on the line shape will be different for the different transitions and pressures whereas the strong interaction width will remain the same. Monte Carlo simulations have shown that a suitable fitting procedure will allow to reach an accuracy in the width determination of  $3 \times 10^{-2}$ .

In a second step a simultaneous measurement of muonic and pionic hydrogen X-rays will be performed. The muonic hydrogen X-rays are affected by Doppler effect in a similar way as their pionic counterparts. In contrast to them they are not broadened by strong interaction, however, and therefore offer a good chance to determine the influence of the Doppler effect [7]. A further check of the theory of the cascade processes will be carried out by measuring the velocity of the system at the instant of the charge exchange process by the neutron time of flight method. The pionic X-ray measurements will start end of year 2000 and first results can be expected in the year after.

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# Pion-kaon scattering

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Pion-kaon scattering has been considered in refs.[1,2] in the framework of three flavor chiral perturbation theory to one loop accuracy. To that order, all low energy constants have been determined from other reactions (and invoking large  $N_c$  arguments), so that phase shifts, threshold and subthreshold parameters can be predicted. Despite the rather large threshold energy,  $\sqrt{s_{\text{thr}}} = M_K + M_\pi \approx 633$  MeV, the one loop corrections to the S-wave scattering lengths are about 25%. The uncertainties due to the various parameters at next-to-leading order are also discussed in [1]. Other approaches and extensions (explicit resonances [3], heavy kaon approach [4], the inverse amplitude method [5] extended to coupled channels [6]) all lead to very similar threshold parameters. The experimental values scatter over large ranges, see e.g. the figure in [2]. Also, two groups have performed a dispersion theoretical analysis [7,8]. While  $a_0^{3/2}$  and  $a_1^{1/2}$  agree with the chiral predictions, the dispersion relation result for  $a_0^{1/2}$  is sizeably larger. A new dispersive analysis including also the SLAC data from 1988 is in progress [9]. In that context, we have also derived a sum rule,

$$\frac{1}{F_K F_\pi} = \frac{4}{\pi} \int_{(M_\pi + M_K)^2}^{\infty} ds' \frac{\text{Im } F^-(s', t)}{(s' - s)(s' - u)}. \quad (1)$$

Even if one assumes that  $\text{Im } F^-$  is entirely given by the  $K^*(892)$  and approximating the latter by a  $\delta$ -function, one obtains  $F_K F_\pi \approx (127 \text{ MeV})^2$  not far off the empirical value of  $(102 \text{ MeV})^2$ . Other work in progress contains a one loop analysis of  $\pi^- K^+ \rightarrow \pi^0 K^0 \gamma$  [10], which is needed to calculate the width of  $\pi K$  atoms. Finally, an analysis of  $\pi K$  scattering in GCHPT is also available [11].

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# $K_{l4}$ decays at DAΦNE

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The capabilities of the KLOE detector [1], and the over constrained kinematic of the  $\phi$ 's decays at DAΦNE [2] will give the opportunity to improve our knowledge on the  $K_{l4}$  decays.

The experimental results obtained so far are dominated by the work of L. Rosselet et al. [3] which measures the  $\pi^+\pi^-$  phase shifts with  $\simeq 30000 K_{e4}^+$  events collected. The phase shift difference  $(\delta_0^o - \delta_1^1)$  was determined in five bins of the invariant mass  $s_\pi$  of the dipion system, and the related isoscalar S-wave scattering length was  $a_0^o = 0.26 \pm 0.05$ . This result must be compared with the CHPT prediction  $a_0^o = 0.20 \pm 0.01$  [4]. The apparent discrepancy between the CHPT prediction and the L. Rosselet experiment may be interpreted as a manifestation of an unusual low value of the quark-antiquark condensate: a new measurement of the isoscalar S-wave scattering length  $a_0^o$  with a substantially smaller error can determine if this discrepancy is statistically significant.

Two methods have been proposed to measure the isoscalar S-wave scattering length  $a_0^o$ : the Pais Treiman method [5], and the Maximum Likelihood Method (*MLM*) [6]. The *MLM* requires to do approximations/assumptions on the form factors, while the Pais Treiman method is model independent. The two methods are complementary and both will be considered at KLOE. Moreover, since the Pais Treiman method was devised for a detector with a uniform efficiency over the whole phase space such as we expect KLOE to be, this will be the first time that this method will be applied (a variant of the Pais Treiman method has been used by L. Rosselet et al. [3]).

The statistical errors on the  $K_{e4}^\pm$  parameters has been evaluated using the *MLM* method [6], in particular, concerning the  $\pi\pi$  phase shifts, the error on the isoscalar S-wave scattering length has been estimated using the J.L. Basdevant et al. parametrisation [7]. For 300000  $K_{e4}^\pm$  events the estimated error is  $\delta a_0^o = 0.01$ . This estimate is purely statistical and apply to a “perfect” detector, i.e., one which covers the whole phase space with unity efficiency everywhere. This is close to being true for KLOE, that it is a hermetic detector operating at DAΦNE producing self-tagging  $K^\pm$  pairs, with high reconstruction efficiency of neutral and charged low energy particles that will be controlled at the level of  $\simeq 10^{-3} \div 10^{-4}$ .

In one year of running at DAΦNE at the target luminosity of  $\mathcal{L}_o = 5 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ , given that the cross section for  $e^+e^- \rightarrow \phi$  at the  $\phi$  resonance peak is  $\simeq 3.8\mu\text{b}$ , the number of  $\phi$  decays into charged kaons will be:

$$N(\phi \rightarrow K^+K^-) \simeq 9.4 \times 10^9$$

and the number of  $K_{e4}$  decays are:

$$N(K_{e4}^\pm) = \underbrace{1.88 \times 10^{10}}_{N(K^\pm)} \times \underbrace{3.9 \times 10^{-5}}_{Br(K_{e4}^\pm)} \times \underbrace{.60}_{\epsilon_{K^\pm}} \times \underbrace{.95}_{\epsilon_{minv}^{cut}} \simeq 4.2 \times 10^5$$

including the efficiency  $\epsilon_{K^\pm}$  to select the  $\phi$  decays in charged kaon events, and the cuts ( $\epsilon_{minv}^{cut}$ ) on the Dalitz invariant mass plot. To collect 300000  $K_{e4}^\pm$  decays in one year, the reconstruction efficiency of the  $K_{e4}^\pm$  events has to be  $\simeq 71\%$ .

The KLOE detector at the Frascati  $\phi$ -factory DAΦNE is fully operational and has recently started to collect its first data. During a short test run period 220 nb $^{-1}$  of integrated luminosity has been accumulated and has been used to study the detector performance and to fully test the reconstruction program. The detector is ready to collect and reconstruct data at the expected luminosity.

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# A new measurement of the $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$ decay.

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The measurement was performed by experiment E865 at the Brookhaven Alternating Gradient Synchrotron employing an apparatus that has been described in previous publications [1]. The selected  $K_{e4}$  candidates were required to have an unambiguous identification of the  $\pi^-$  to suppress background originating from  $K^+$  decays with a  $\pi^0$  (mostly  $K_{\pi 2}$ ,  $K_{e3}$ , and  $K_{\mu 3}$ ), giving an  $e^+e^-$  pair. In order to reject the  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  ( $K_\tau$ ) background, we require an unambiguous identification of the  $e^+$  and a kaon, reconstructed from the three charged decay products, which does not track back to the target. The basis of the rejection of accidental background is the requirement to have three trajectories consistent with having come from a common vertex, and a timing spread between the trajectories consistent with the resolution. Our final signal sample contains 437,000  $K_{e4}$  candidates including 2% background events. These data represent a more than tenfold increase in statistics compared with previous experiments [2]. Fig. 3a) shows the reconstructed  $\pi\pi$  mass in comparison with the Monte Carlo simulation, one of many control plots, demonstrating our good understanding of the data.

These data were then used to fit phase shift differences and form factors as a function of the  $\pi\pi$  mass. In a first analysis, a formalism as it was described by Rosselet [2] was employed. Radiative corrections were not included but their calculation is in progress [3]. A preliminary fit of the phase shift differences as a function of the  $\pi\pi$  mass is shown in Fig. 3b). An estimation of the phase shifts and form factors based on the parameterization proposed by Amorós and Bijmans [4] and the determination of the branching ratio is in progress.

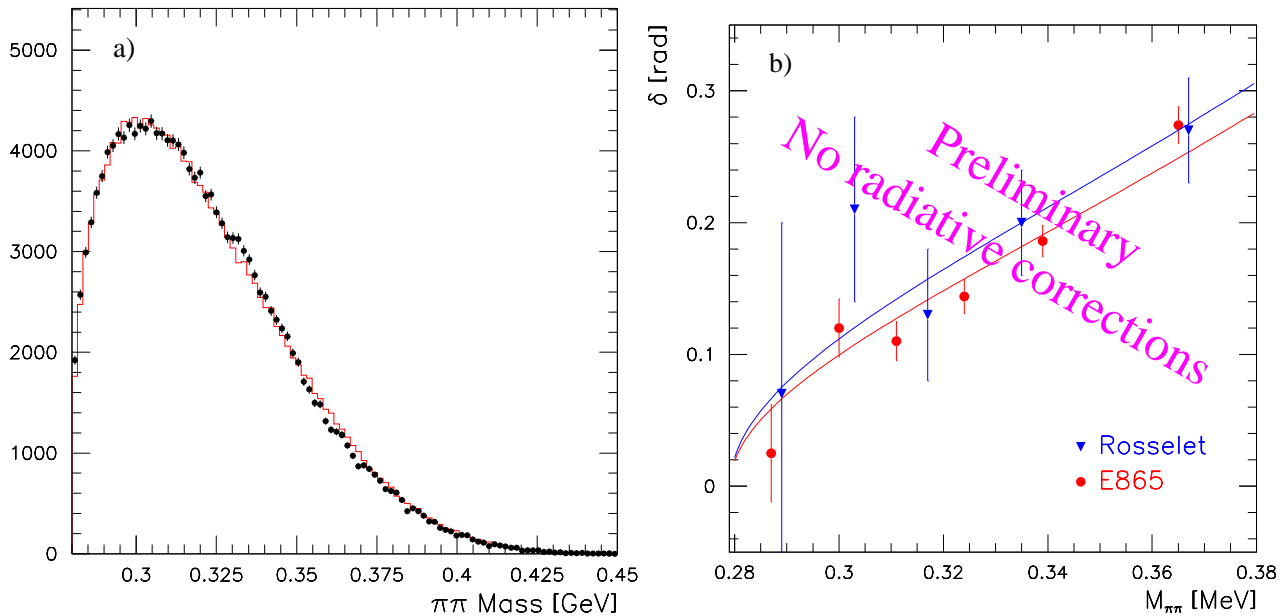


Figure 3: a) Distribution of the reconstructed  $\pi\pi$  mass. The dots represent the data, and the histogram shows a Monte Carlo simulation. b) Dependence of the phase shift difference  $\delta \equiv \delta_0^0 - \delta_1^1$  on the  $\pi\pi$  mass. The fits are based on the work by Basdevant *et al.* [5].

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## $K_{\ell 4}$ decays: a theoretical discussion

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To measure the s-wave of the  $\pi\pi$  scattering, the experiments need to focus on processes where the inelastic channels are suppressed. The decay of the pionium system and the  $K_{\ell 4}$  decays are competitive and independent ways. This talk will be related with  $K_{\ell 4}$  decays where the supposed precision for the experiment will force the calculation to higher orders in CHPT [1]. An overview of the status of the theoretical calculation and the influence on the experimental parametrization [2] is given. A first part of the talk is related with a general and model independent parametrization for the  $K_{\ell 4}$  decays. Previous study of the prediction for the relevant form factors in several models and in the framework of CHPT allows us to consider the linear dependence with the kinematical variables as the natural one. This parametrization could decrease the number of variables to fit. In the second part of the talk some results about the two-loops calculation is given. Decay constants and masses for the complete  $SU(3)$  octet are obtained.

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# Chiral phase transitions at zero temperature

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It is argued that in QCD , quark loops progressively suppress order parameters of chiral symmetry breaking as the number  $N_f$  of light flavours increases. This could result into a non-trivial chiral phase structure of the theory considered as a function of  $N_f$  and  $N_c$ . For  $N_f$  close to the first critical point  $n_{crit}(N_c)$ , one may expect [1] a reduction of the size of the chiral condensate  $\langle \bar{q}q \rangle$  and an enhancement of its fluctuations leading to an important violation of the Zweig rule (i.e. to a breakdown of large  $N_c$  predictions) in the scalar iso-scalar channel. Starting from observed correlation between  $\bar{u}u$  and  $\bar{s}s$ , it is possible to estimate the fall-off of the condensate  $\langle \bar{u}u \rangle_{N_f}$  as the number of light spectators varies from  $N_f = 2$  to  $N_f = 3$  [2]. Existing data are consistent with a strong  $N_f$  - dependence : The three- flavour condensate could be suppressed by as much as a factor 2 relative to the two-flavor condensate, possibly indicating that the real world ( $N_f = 2 - 3, N_c = 3$ ) could already feel the influence of a nearby phase transition [2]. It is important to make a careful distinction between  $\langle \bar{u}u \rangle_2$  and  $\langle \bar{u}u \rangle_3$  in all phenomenological analysis. The two-flavour condensate (in appropriate units) is measurable in precise low-energy  $\pi - \pi$  scattering experiments and is strongly correlated with the quark mass ratio  $r = 2m_s/(m_u + m_d)$  [1]. The main goal of these experiments remains the determination of the two-flavour condensate, in order to examine to which extent the expected decrease of the condensate  $\langle \bar{u}u \rangle_{N_f}$  towards the critical point already starts to be visible at  $N_f = 2$ . Whatever the experimental answer to this question will be, additional experimental information will be necessary to pin down the three-flavour condensate and to settle the theoretical issue of a nearby phase transition as a common explanation of the observed large Zweig rule violation in the vacuum-channel and of the  $N_f$  dependence of the chiral symmetry breaking. This might require new experiments involving Kaons ,in which the strange quark would play a more direct role than that of a “sea-side-spectator”.

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# The two-flavor chiral condensate from low-energy $\pi\pi$ phase-shifts

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Low-energy  $\pi\pi$  scattering has been identified as the most promising physical observable for measuring the quark condensate [1]. What can actually be measured in  $\pi\pi$  experiments is the two-flavor quark condensate, i.e.  $\langle\bar{q}q\rangle$  in the  $SU(2)\times SU(2)$  chiral limit [2]. Due to the properties of analyticity, unitarity and crossing symmetry and to the fact that Goldstone bosons interact weakly at low energy, the amplitude  $A(s|, t, u)$  can be written, up to and including the two-loop level, in terms of six parameters  $(\alpha, \beta, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ . This low-energy representation holds independently of the size of  $\langle\bar{q}q\rangle$ . The explicit expression can be found in Ref. [3].  $\alpha$  and  $\beta$  represents essentially the amplitude and its slope at the symmetrical point  $s = t = u = 4/3M_\pi^2$ . They can be directly related to the two-flavor quark condensate measured by the Gell-Mann–Oakes–Renner ratio  $x_2 = -2\hat{m} \lim_{\hat{m} \rightarrow 0} \langle\bar{q}q\rangle / (F_\pi^2 M_\pi^2)$ . In addition to constraints from chiral symmetry, the amplitude satisfies also a set of crossing-symmetrical dispersion relations, known as Roy Equations. Using as input experimental phase-shifts in the medium- and high-energy region, they allow to determine the low-energy amplitude in terms of two subtraction constants, usually identified with the two scalar scattering lengths  $a_0^0$  and  $a_0^2$ . In fact the two subtraction constants can be related to each other, imposing the matching of the amplitude to the experimental  $P$ -wave, which is well known down to rather low energies, thanks to the  $\rho$  resonance. This results in the so-called Morgan-Shaw universal curve, that we quote in the form given by Petersen in 1979,

$$2a_0^0 - 5a_0^2 = 0.692 \pm 0.027 + 0.9(a_0^0 - 0.3) + 1.2(a_0^0 - 0.3)^2.$$

By comparing the two low-energy representations one is able, in principle, to determine 4 out of the 6 parameters of the chiral amplitude. A similar program has been undertaken in Ref. [3], in order to fix the parameters  $\lambda_1, \dots, \lambda_4$ . It turns out that the outcome of this analysis is only barely dependent on  $\alpha$  and  $\beta$ , and hence on the quark condensate. The slight  $\alpha$ - and  $\beta$ -dependence can be conveniently approximated by linear functions  $\lambda_i^{\text{exp}}(\alpha, \beta)$ ,

$$\begin{aligned} \lambda_1^{\text{exp}} &= [-6.08 - 0.37(\alpha - 2) + 6.8(\beta - 1.08)] \cdot 10^{-3}, & \lambda_2^{\text{exp}} &= [9.56 + 0.22(\alpha - 2) + 2.2(\beta - 1.08)] \cdot 10^{-3}, \\ \lambda_3^{\text{exp}} &= [2.20 - 0.01(\alpha - 2) + 1.1(\beta - 1.08)] \cdot 10^{-4}, & \lambda_4^{\text{exp}} &= [-1.46 + 0.02(\alpha - 2) - 1.6(\beta - 1.08)] \cdot 10^{-4}, \end{aligned}$$

while the experimental uncertainties on the  $\lambda_i$  are independent of  $\alpha$  and  $\beta$ ,  $\delta\lambda_1^{\text{exp}} = 2.2 \cdot 10^{-3}$ ,  $\delta\lambda_2^{\text{exp}} = 0.5 \cdot 10^{-3}$ ,  $\delta\lambda_3^{\text{exp}} = 0.6 \cdot 10^{-4}$ ,  $\delta\lambda_4^{\text{exp}} = 0.12 \cdot 10^{-4}$ . With the parameters  $\lambda_1, \dots, \lambda_4$  determined from the Roy dispersion relations, the low-energy  $\pi\pi$  observables are effectively parametrized only by  $\alpha$  and  $\beta$ , which can be fitted to the experimental phase-shifts. However, in order to fully exploit the analytical properties of the amplitude contained in the two-loop six-parametrical formula of Ref. [3], we can treat all the six parameters as fit variables, and add the  $\lambda_i^{\text{exp}}(\alpha, \beta)$  as additional experimental points to the  $\chi^2$ . Moreover one can insert the constraint of the Morgan-Shaw universal curve, which, using the two-loop expressions for the scattering lengths [3], can also be expressed in terms of the fit variables  $(\alpha, \beta, \lambda_1, \dots, \lambda_4)$ . Thus we define the  $\chi^2$  as

$$\chi^2 = \sum_n \left[ \frac{\delta_n(\alpha, \beta, \lambda_1, \lambda_2, \lambda_3, \lambda_4) - \delta_n^{\text{exp}}}{\sigma_n^{\text{exp}}} \right]^2 + \sum_{i=1}^4 \left[ \frac{\lambda_i - \lambda_i^{\text{exp}}(\alpha, \beta)}{\delta\lambda_i^{\text{exp}}} \right]^2 + \chi_{\text{Morgan-Shaw}}^2(\alpha, \beta, \lambda_1, \lambda_2, \lambda_3, \lambda_4),$$

where  $\delta_n^{\text{exp}}$  are the experimental phase-shifts, with experimental error  $\sigma_n^{\text{exp}}$ . It is straightforward to include to the  $\chi^2$  the combination  $a_0^0 - a_0^2$ , once the pionium lifetime will be measured. Applying this fit procedure to the old  $K_{e4}$  Rosselet data [ $\delta_n^{\text{exp}} = \delta_0^0(E_n) - \delta_1^1(E_n)$ ], we find for the  $S$ -wave isoscalar scattering length,  $a_0^0 = 0.268 \pm 0.043$ , and for the combination relevant to the pionium lifetime,  $a_0^0 - a_0^2 = 0.294 \pm 0.033$ . Notice that the error on the difference of scattering lengths is smaller than the individual error on  $a_0^0$ , reflecting the importance of properly taking into account all the correlations among the fit variables. Using the relationship between  $x_2$  and  $(\alpha, \beta)$ , known at one-loop level, we get  $x_2 = 0.71 \pm 0.13_{\text{exp}} \pm 0.05_{\text{theo}}$ , where the experimental and theoretical errors can be considered as mutually uncorrelated.

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# Radiative corrections to semi-leptonic decays

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Semi-leptonic decays are an important source of information on hadronic matrix elements of the Standard Model vector and axial currents and, in the case of the  $K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e$  decay, on low-energy  $\pi - \pi$  scattering. In order to meet the accuracy of two-loop calculations and the expected improvements on the experimental side in the near future (BNL-E865, KLOE, DIRAC,...), electromagnetic corrections to these processes, or isospin breaking contributions in general, have to be estimated in a reliable way.

The first step towards this aim has been proposed in Ref. [1]. In order to address the question of electromagnetic corrections in semi-leptonic processes on a systematical basis, the particle spectrum of chiral perturbation theory with virtual photons has been enlarged by including also the light leptons. The presence of radiative corrections affects the semi-leptonic amplitudes in two ways. First, the factorization property between the hadronic and the leptonic weak currents is lost, due to loop graphs involving both virtual photons and virtual leptons. Second, these additional loop graphs generate in general new divergences, which are taken care of neither by the Gasser-Leutwyler low-energy constants  $L_i$  [2], nor by Urech's counterterms  $K_i$  [3]. The structure of these divergences have been analysed at one loop, using super-heat-kernel methods [4], and a complete list of the corresponding counterterms has been given in [1]. As an illustration of this framework, an explicit calculation of the decay rates for  $\pi \rightarrow \ell \nu(\gamma)$  and  $K \rightarrow \ell \nu(\gamma)$  has also been provided. The extensions to the other semi-leptonic processes, and to the pion  $\beta$ -decay, are in progress.

A numerical estimate of the corresponding decay rates eventually requires a determination of the counterterms involved in each amplitude. This may be achieved following methods originally suggested in Ref. [5], combined with large- $N_C$  techniques [6]. From this point of view, the recent study [7] of the decay of pseudoscalar mesons into pairs of charged leptons, which, as far as some aspects of this question are concerned, may be considered as a “weak SU(2) rotation” of the  $\pi \rightarrow \ell \nu$  decay, offers interesting perspectives.

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# Numerical solutions of Roy equations

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We solve numerically the Roy equations [1] for the  $S$  and  $P$  waves of the  $\pi\pi$  scattering amplitude in the low-energy region (below 800 MeV) [2]. The imaginary parts above 800 MeV or of higher waves are taken either from experiments (where available) or from theoretical modelling. We are able to solve the equations for any  $S$  wave  $I = 0$  scattering length  $a_0^0$ . At fixed  $a_0^0$  solutions can be found only for  $a_0^2$  (the  $S$  wave  $I = 2$  scattering length) within a certain range, called the Universal Band.

We then confront the solutions we find for various values of  $a_0^0$  and  $a_0^2$  to the available experimental data. The corresponding  $\chi^2$  analysis yields ellipses of given confidence level inside the Universal Band. We confirm old analyses [3] of the  $K_{e4}$  data [4] leading to an  $a_0^0$  between 0.20 and 0.30 [5]. This machinery is ready to be used with forthcoming  $K_{e4}$  data which should reduce drastically the error on  $a_0^0$ .

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# Analysis of bound state problems with non relativistic Lagrangians

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We discuss the use of Non Relativistic Lagrangians for the study of bound state characteristics and in particular of hadronic atoms. This study can give important information about strong interactions at low energies. Hence it has been the main aim of many experiments performed in the last years and of others, like DIRAC at CERN and DEAR at DAΦNE, which are running now or planned to start soon. The nature of the problem (and mainly the fact that the bound state components momenta are much smaller than their masses) suggests the use of non relativistic lagrangians. This approach, adopted in a number of recent publications [1-2], has proved to be probably the most efficient way to study these systems in quantum field theories. In this connection, we feel that it is useful to discuss general properties of the non relativistic framework in a detailed and systematic manner. We refer in particular to the correct matching and regularization of the theory and to the problem of ensuring the UV finiteness of the bound state characteristics and their independence on the off-shell behaviour of the Green functions.

We found it worthwhile to study these problems by using an appropriate model. Our results[3], that will appear soon, have proved the possibility of developing a consistent quantum field theoretical framework for studying bound states with non relativistic lagrangians. Our model has a massive scalar bound in an external coulombic field and interacting with another massless scalar field. It has the advantage of being exactly solvable and it presents all the essential features of the physical systems one wants to study, without being affected by some inessential complications. We have studied this model at different levels of complexity.

Neglecting, at first stage, the interaction with the massless scalar, one computes the relativistic wave functions and energy levels of the heavy scalar in the external field, by looking for the poles of the Fourier transform of the two point function. In a non relativistic framework one recovers an effective lagrangian starting from the relativistic one and requiring that the S matrix elements are the same in both theories. Then, one can use Rayleigh-Schrödinger perturbation theory to evaluate the energy levels. Our lagrangian, written as an expansion in inverse powers of the heavy mass, is built in such a way that the non relativistic and relativistic Green functions are the same and this guarantees that the correct energy levels are reproduced in the effective theory. The lagrangians obtained with different techniques give the same S matrix elements on mass shell. We have shown that they differ only by terms that can be eliminated by using field redefinitions and that don't contribute to the bound state energies. We have also verified that, even in the effective theories, the divergences appearing in the bound state calculations can be cancelled, by summing all the contributions of a given order in the coupling constant and using the same counterterms needed to make the Green functions finite at the same order.

Finally, we have added an interaction term between the heavy scalar field and a dynamical massless scalar and computed at first order in the coupling constant the energy shift (due to radiative corrections) for the ground state of the heavy scalar. To perform the non relativistic calculation one has to face the problem of the breaking of the naive power counting rules. In, fact computing the self energy <sup>1</sup> in dimensional regularization one sees that the loop contributions fail to reproduce the expansion in powers of the momenta on the basis of which the effective lagrangian is built. So one must impose some additional rules, like the so called "multipole expansion" that we have adopted, recovering an efficient power counting. With this prescription we have verified that the non relativistic energy shift at the first order in the coupling constants reproduces exactly the relativistic one.

To summarize, it is possible to develop a completely consistent and well defined formalism, based on the use of non relativistic lagrangians, to study the hadronic atoms. These techniques already applied successfully to the case of ponium [1-2] could be extended also to the analysis of other exotic atoms (like  $p\pi^-$ ,  $pK^-$ ,  $d\pi^-$ ,  $dK^-$ ).

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<sup>1</sup>The problem of self energy calculation and of a correct definition of the mass in the effective theory will be treated in detail in a separate work [4]